

# Note on the Slope Parameter of the Baryonic

## $\Lambda_b \rightarrow \Lambda_c$ Isgur-Wise Function

Ming-Qiu Huang<sup>1</sup>, Hong-Ying Jin<sup>2</sup>, J. G. Körner<sup>3</sup> and Chun Liu<sup>4</sup>

<sup>1</sup>*Physics Department, Nat'l University of Defense Technology, Changsha 410073, China*

<sup>2</sup>*Institute of Modern Physics, Zhejiang University, Hangzhou 310027, China*

<sup>3</sup>*Institut für Physik, Johannes Gutenberg-Universität, D-55099 Mainz, Germany*

<sup>4</sup>*Institute of Theoretical Physics, Chinese Academy of Sciences,*

*P.O. Box 2735, Beijing 100080, China*

### Abstract

Using the framework of the Heavy Quark Effective Theory we have re-analyzed the Isgur-Wise function describing semileptonic  $\Lambda_b \rightarrow \Lambda_c$  decays in the QCD sum rule approach. The slope parameter of the Isgur-Wise function is found to be  $\rho^2 = 1.35 \pm 0.12$ , which is consistent with an experimental measurement and a lattice calculation. To  $\mathcal{O}(1/m_b, 1/m_c)$  of the heavy quark expansion the integrated  $\Lambda_b$  decay width is used to extract the CKM matrix element  $V_{cb}$  for which we obtain a value of  $|V_{cb}| = 0.041 \pm 0.004$  in excellent agreement with the value of  $|V_{cb}|$  determined from semileptonic  $B \rightarrow D^*$  decays.

The study of  $b \rightarrow c$  semileptonic weak decays has been the subject of considerable interest in recent years, as a source of information on  $V_{cb}$  and as a laboratory for understanding the strong interaction effects and developing nonperturbative QCD methods. A considerable amount of work has been carried out in the meson sector, in which the Heavy Quark Effective Theory (HQET) [1] was first developed. In the baryon sector a particular example is the semileptonic decay  $\Lambda_b \rightarrow \Lambda_c \ell \bar{\nu}_\ell$ . In view of the fact that a recent experiment from DELPHI [2] shows a discrepancy with the results of previous QCD sum rule calculations [3, 4] an updated sum rule analysis for the baryonic  $\Lambda_b \rightarrow \Lambda_c$  form factor is called for.

In the heavy quark limit, the hadronic matrix element of the  $\Lambda_b \rightarrow \Lambda_c$  transition can be simply expressed in terms of a single Isgur-Wise (IW) function defined as follows [5, 6, 7],

$$\langle \Lambda_c(v') | \bar{c} \Gamma b | \Lambda_b(v) \rangle = \xi(\omega) \bar{u}_{\Lambda_c}(v') \Gamma u_{\Lambda_b}(v), \quad (1)$$

where  $\omega = v \cdot v'$  is the velocity transfer variable and  $\Gamma$  is an arbitrary gamma matrix. When the velocity of the heavy quark changes from  $v$  to  $v'$  due to the weak decay of the heavy quark, the light degrees of freedom undergo a corresponding transition due to their strong interactions with the heavy quark. The Isgur-Wise (IW) function  $\xi(\omega)$  is a measure of the transition amplitude of the light degrees of freedom. The IW function  $\xi(\omega)$  is normalized to 1 at the zero recoil point  $\omega = 1$ . This value is reduced by a few percent after taking into account radiative QCD correction effects [8, 9] <sup>1</sup> To obtain a theoretical description of the whole IW function one must use non-perturbative methods which, at the current stage, are beset with large uncertainties. In the decay  $\Lambda_b \rightarrow \Lambda_c \ell \bar{\nu}_\ell$ , the physical region of  $\omega$  lies in the range 1 to 1.43. Usually the IW function is expanded up to the first order in  $(\omega - 1)$ ,

$$\xi(\omega) = 1 - \rho^2(\omega - 1) + \mathcal{O}((\omega - 1)^2) \quad (2)$$

The slope parameter  $\rho^2$  of the IW function at zero recoil has to be calculated using non-perturbative methods.

The QCD sum rule approach [11, 12] has proven to be a reliable tool to deal with many problems in the realm of nonperturbative QCD. It has been used successfully to calculate the properties of various hadrons. For instance, besides the light mesons and baryons, heavy meson properties were systematically analyzed in the sum rule approach within the framework of HQET [8]. In the heavy baryon sector the masses of heavy baryons and the IW functions describing their weak transitions were calculated in Refs. [3, 4, 13, 14] and [15, 16, 17, 18, 19], respectively. In Ref. [20], the calculation for the heavy baryons began with the full theory and the results of the calculation were expanded in terms of

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<sup>1</sup>At zero recoil the  $\Lambda_b \rightarrow \Lambda_c$  weak transition matrix elements have no  $1/m_Q$  corrections, and the  $1/m_Q^2$  corrections are small.

inverse powers of the heavy quark masses. In the HQET sum rule approach the baryonic  $\Lambda_b \rightarrow \Lambda_c$  IW function was calculated in [3, 4], and the slope parameter  $\rho^2$  was fitted to lie in the range  $0.5 - 0.8$ . However, such low slope values would predict exclusive  $\Lambda_b \rightarrow \Lambda_c$  semileptonic decay rates dangerously close to the inclusive semileptonic rate [21, 22]. A first measurement of the IW function of semileptonic  $\Lambda_b \rightarrow \Lambda_c$  transitions has recently been reported by the DELPHI Collaboration [2]. The errors on this measurement are quite large. Using an exponential parametrisation, they quote a value of  $\rho^2 = 1.59 \pm 1.10(\text{stat})$ . When the observed event rates were included in the fit they obtained  $\rho^2 = 2.03 \pm 0.46(\text{stat})_{-1.00}^{+0.72}(\text{syst})$  [2]. Within the large error bars the experimental slope value is compatible with the HQET sum rule results of [3, 4] although the experimental central values of [2] are considerably higher than the theoretical sum rule results.

Theoretically, the above-mentioned HQET sum rule results for the slope parameter  $\rho^2 = 0.5 - 0.8$  appear to be rather small. Because the number of light quark transitions is larger in the heavy baryon case than in the heavy meson case one expects that the slope of the baryonic IW function is larger than that of the mesonic IW function. In fact, in the large  $N_c$  limit,  $\rho^2$  will be infinitely large [23]. In the spectator quark model approach [6, 7, 24] one finds  $\rho^2 = 2\rho_{\text{meson}}^2 - 1/2$  which turns into an upper bound  $\rho^2 \leq 2\rho_{\text{meson}}^2 - 1/2$  when the interaction between the light quarks is turned on [25]<sup>2</sup>.

Concerning the slope parameter of the mesonic IW function, one finds theoretical values of about 1 from sum rule calculations [8]. Experimental numbers for the mesonic slope parameter also scatter around 1 [26, 27, 28]. Using the spectator quark model estimate one thus expects baryonic values of the slope parameter  $\rho^2$  in the vicinity of 1.5 or slightly below that number. The Skyrme model predicts  $\rho^2 \simeq 1.3$  [29]. In the infinite momentum frame model one has  $\rho^2 = 1.44$  [18] and in the relativistic three quark model one finds  $\rho^2 = 1.35$  [19]. For the baryonic sum rule results, radiative corrections to  $\rho^2$  and  $1/m_Q$  corrections to the form factors are not expected to be large enough to solve the discrepancy between the large experimental central value for  $\rho$  in [2] and the small QCD sum rule results [3, 4]. We therefore concentrate only on the leading order results in our analysis.

The purpose of this work is to present a new QCD sum rule analysis for the leading-order Isgur-Wise function describing the  $\Lambda_b \rightarrow \Lambda_c$  transition. In particular, we concentrate on the sum rule prediction directly for the slope parameter  $\rho^2$ . To start with, we first review the sum rule analysis of the two-point Green's function involving two heavy baryon currents relevant for the determination of the heavy baryon decay constant and its mass. A possible choice of the heavy baryon current with the correct quantum numbers of the

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<sup>2</sup>The large  $N_c$  result in the spectator quark model approach can be worked out to be  $\rho^2 = (N_c - 1)(\rho_{\text{meson}}^2 - 1/4)$ . Since  $\rho_{\text{meson}}^2 \geq 1/2$  according to the sum rule of Bjorken, one then recovers the infinite slope result of [23] in the large  $N_c$  limit.

heavy baryon  $\Lambda_Q$  is given by

$$\tilde{j}^v = q^T C \tau \gamma_5 q h_v, \quad (3)$$

where  $C$  is the charge conjugation matrix,  $\tau$  is an antisymmetric flavor matrix, and  $h_v$  and  $q$  are the heavy and light quark fields. Note that there is an alternative choice for the heavy baryon current  $\tilde{j}^v$  which is obtained by the replacement  $\gamma_5 \rightarrow \gamma_5 \not{v}$  in Eq.(3). The two baryon currents give the same diagonal sum rule for the IW function.

From the correlator

$$\Gamma(\epsilon) = i \int d^4x e^{ikx} \langle 0 | T \{ \tilde{j}^v(x) \bar{\tilde{j}}^v(0) \} | 0 \rangle \quad (4)$$

one can obtain the two-point sum rule [3, 4, 13, 14]:

$$2f^2 e^{-\bar{\Lambda}/T} = \frac{1}{10\pi^4} \int_0^s d\nu \nu^5 e^{-\nu/T} + \frac{\langle \bar{q}q \rangle^2}{3} e^{-\frac{m_0^2}{sT^2}} + \frac{\langle \alpha_s GG \rangle}{16\pi^3} T^2, \quad (5)$$

where the baryonic decay constant is defined as  $\langle 0 | \tilde{j}^v | \Lambda_Q \rangle \equiv fu$ .  $\bar{\Lambda} = m_{\Lambda_Q} - m_Q$  is the binding energy of the  $\Lambda_Q$ -baryon in HQET.  $T$  is the Borel parameter. According to the duality assumption the higher resonance and continuum contributions to the Green's function are approximated by the perturbative contribution above a given threshold  $s$ . Note that the factor 2 on the l.h.s. of Eq. (5) is missing in the calculation of Ref.[15]. The missing factor 2 is, however, of no relevance for the calculation of  $\bar{\Lambda}$  and the IW function.

To obtain the IW function, one considers the three-point correlator

$$\tilde{\Xi}(\epsilon, \epsilon', \omega) = i^2 \int d^4x_1 d^4x_2 e^{i(k'x_1 - kx_2)} \langle 0 | T \tilde{j}^{v'}(x_1) \bar{h}_{v'}(0) \Gamma h_v(0) \bar{\tilde{j}}^v(x_2) | 0 \rangle, \quad (6)$$

where  $\epsilon = v \cdot k$  and  $\epsilon' = v' \cdot k'$ . In the hadronic language the three-point function Eq. (6) can be expressed in terms of insertions of hadronic states. The lowest contribution involves the IW function,

$$\tilde{\Xi}(\epsilon, \epsilon', \omega) = \frac{f^2 \xi(\omega)}{(\bar{\Lambda} - \epsilon)(\bar{\Lambda} - \epsilon')} \frac{1 + \not{v}'}{2} \Gamma \frac{1 + \not{v}}{2} + \text{resonances}. \quad (7)$$

On the other hand, the three-point correlator  $\tilde{\Xi}$  can be calculated by using the Operator Product Expansion. The perturbative contribution can be written in terms of the double dispersion relation,

$$\tilde{\Xi}^{\text{pert}}(\epsilon, \epsilon', \omega) = \int_0^\infty d\tilde{\epsilon} \int_0^\infty d\tilde{\epsilon}' \frac{\varrho(\tilde{\epsilon}, \tilde{\epsilon}', \omega)}{(\tilde{\epsilon} - \epsilon)(\tilde{\epsilon}' - \epsilon')}, \quad (8)$$

with the following spectral density,

$$\varrho(\epsilon, \epsilon', \omega) = \frac{3}{26\pi^4} \frac{1}{(\omega^2 - 1)^{5/2}} (\epsilon^2 + \epsilon'^2 - 2\omega\epsilon\epsilon')^2 \text{tr}(\tau\tau^\dagger) \frac{1 + \not{v}'}{2} \Gamma \frac{1 + \not{v}}{2} \Theta(\epsilon) \Theta(\epsilon') \Theta(2\omega\epsilon\epsilon' - \epsilon^2 - \epsilon'^2). \quad (9)$$

The condensate contributions will be included as a series of vacuum expectation values of operators ordered by their dimension.

A systematic uncertainty of QCD sum rule calculations lies in the treatment of higher state contributions to the hadronic side of  $\tilde{\Xi}$ . Generally the quark-hadron duality assumption is adopted, which simulates the higher state contribution by the perturbative part above some threshold energy. For three-point Green's functions, this assumption is more ambiguous than that for the two-point case because there are two energy variables  $\tilde{\epsilon}$  and  $\tilde{\epsilon}'$ . It was argued by Blok and Shifman in [30], that the perturbative and the hadronic spectral densities cannot be locally dual to each other. The necessary way to restore duality is to integrate the spectral densities over the “off-diagonal” variable  $\tilde{\epsilon}_- = (\tilde{\epsilon} - \tilde{\epsilon}')/2$ , keeping the “diagonal” variable  $\tilde{\epsilon}_+ = (\tilde{\epsilon} + \tilde{\epsilon}')/2$  fixed. It is w.r.t.  $\tilde{\epsilon}_+$  that the quark-hadron duality is assumed for the integrated spectral densities. With this procedure the sum rule for the IW function yields [16],

$$2f^2\xi e^{-\bar{\Lambda}/T} = \frac{4}{5\pi^4} \frac{1}{(\omega+1)^3} \int_0^{\tilde{s}} d\tilde{\epsilon}_+ \tilde{\epsilon}_+^5 e^{-\tilde{\epsilon}_+/T} + \frac{\langle\bar{q}q\rangle^2}{3} e^{-\frac{m_0^2}{16T^2}(\omega+1)} + \frac{\langle\alpha_s GG\rangle}{12\pi^3} T^2 \frac{2\omega+1}{(\omega+1)^2}, \quad (10)$$

where  $\tilde{s}$  is the threshold energy for the sum rule. It should be noted that the Borel parameter has been chosen such that  $\xi(\omega=1)=1$ . The  $\omega$  dependence in the gluon condensate term and the coefficient of the exponential in the quark condensate term are different from Ref. [15], but are consistent with Refs. [17, 20]. From Eq. (10) one obtains the sum rule for the slope parameter of the IW function, i.e.  $\rho^2 \equiv -\frac{d\xi}{d\omega}|_{\omega=1}$ . It is given by

$$2f^2\rho^2 e^{-\bar{\Lambda}/T} = \frac{3}{20\pi^4} \int_0^{\tilde{s}} d\tilde{\epsilon}_+ \tilde{\epsilon}_+^5 e^{-\tilde{\epsilon}_+/T} + \frac{m_0^2\langle\bar{q}q\rangle^2}{48T^2} e^{-\frac{m_0^2}{8T^2}} + \frac{\langle\alpha_s GG\rangle}{48\pi^3} T^2. \quad (11)$$

The baryonic decay constant  $f$  can be obtained by the sum rule Eq. (5). Note that we have not included the perturbative  $\mathcal{O}(\alpha_s)$  corrections in the sum rule which is expected to be largely cancelled in the ratio of Eqs. (10) and (5), or Eqs. (11) and (5). This is what happened in the case of heavy meson form factors [8].

For the numerical analysis of the sum rule (11) we use the two-point sum rule (5) to eliminate the explicit dependence of Eq. (11) on  $f$  and  $\bar{\Lambda}$ . This procedure reduces the uncertainties in the calculation. For the condensate contributions we take the standard values

$$\langle\bar{q}q\rangle = -(0.24 \pm 0.01)^3 \text{ GeV}^3, \quad \langle\frac{\alpha_s}{\pi} GG\rangle = (0.012 \pm 0.004) \text{ GeV}^4 \quad (12)$$

and  $m_0^2 = 0.8 \text{ GeV}^2$ . The threshold energies are taken to be equal, i.e.  $s = \tilde{s} = s_c$ . Imposing the usual criterion on the ratio of contributions of the higher-order power corrections and that of the continuum and using the central values of the condensates given in (12) there is an acceptable window of stability in the range  $T = 0.4 - 0.7 \text{ GeV}$  in which the

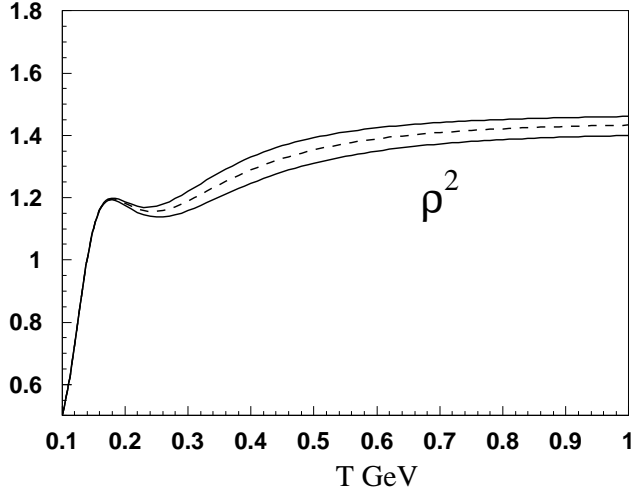


Figure 1: The slope parameter of the IW function  $\rho^2$  versus the Borel variable  $T$ . The curves refer to choices of the threshold parameters:  $s_c = 2.1$ ,  $s_c = 1.9$ ,  $s_c = 1.7$ , from top to bottom.

calculated results do not change appreciably if the threshold parameter  $s_c$  lies in the range  $1.7 < s_c < 2.1$  GeV. In Fig. 1, the sum rule for the slope parameter of the IW function  $\rho^2$  is plotted as a function of the Borel parameter  $T$  for various choices of the continuum threshold in the range  $1.7 < s_c < 2.1$  GeV. One can see that the variation is very moderate for the Borel parameter in the range  $0.4 < T < 0.7$  GeV. Our prediction for the slope parameter  $\rho^2$  is given by

$$\rho^2 = 1.35 \pm 0.12. \quad (13)$$

The errors reflect the uncertainty due to the sum rule window. The value of the slope parameter is in agreement with the recent experimental result [2] and the value obtained in a lattice determination [31].

It is meaningful to ask why the present sum rule result is so different from those obtained previously. First, this maybe due to the systematic uncertainty of QCD sum rules. The systematic error resulting from the use of quark-hadron duality above  $s_c$  is difficult to estimate. Conservatively speaking, there is a 10% – 30% systematic error. Second, the previous linear fit to the IW function [16, 15] may be a rather poor fit in the  $\omega$  range: 1 – 1.43. In the following decay rate calculation, we assume an exponential form to parametrize the baryonic IW function, as the experiment did [2]:

$$\xi(\omega) = \xi(1) \exp[-\rho^2(\omega - 1)]. \quad (14)$$

Once we have computed the IW function we are now in the position to calculate the rate for the semileptonic  $\Lambda_b \rightarrow \Lambda_c \ell \bar{\nu}_\ell$  transitions. Neglecting the lepton mass the differential

decay rate can be written as (see e.g. [4, 7])

$$\begin{aligned} \frac{d\Gamma}{d\omega} = & \frac{G_F^2 |V_{cb}|^2 m_{\Lambda_b}^5 r^3}{24\pi^3} \sqrt{\omega^2 - 1} \left\{ (\omega - 1) \left[ 2\kappa F_1^2 + [(1+r)F_1 + (\omega+1)(rF_2 + F_3)]^2 \right] \right. \\ & \left. + (\omega + 1) \left[ 2\kappa G_1^2 + [(1-r)G_1 - (\omega-1)(rG_2 + G_3)]^2 \right] \right\}, \end{aligned} \quad (15)$$

where  $r = m_{\Lambda_c}/m_{\Lambda_b}$  and  $\kappa = 1 + r^2 - 2r\omega$ . The form factors  $F_i$  and  $G_i$  can be expressed by a set of Isgur-Wise functions at each order in  $1/m_Q$  in HQET. Taking into account the  $1/m_Q$  corrections, they are (see e.g. [4, 7])

$$\begin{aligned} F_1 &= [1 + (\varepsilon_c + \varepsilon_b)\bar{\Lambda}] \xi(\omega), & F_2 &= G_2 = -\frac{2\varepsilon_c\bar{\Lambda}}{\omega+1} \xi(\omega), \\ G_1 &= [1 + (\varepsilon_c + \varepsilon_b)\bar{\Lambda}\frac{\omega-1}{\omega+1}] \xi(\omega), & F_3 &= -G_3 = -\frac{2\varepsilon_b\bar{\Lambda}}{\omega+1} \xi(\omega), \end{aligned} \quad (16)$$

where  $\varepsilon_Q = 1/(2m_Q)$ . Notice that the subleading Isgur-Wise function associated with the insertion of the  $\Lambda_{\text{QCD}}/m_c$  kinetic operator of the HQET Lagrangian has been neglected since it is negligibly small [16, 18]. In the numerical calculation we take the heavy quark masses to be  $m_b = 4.8$  GeV,  $m_c = 1.4$  GeV and  $\bar{\Lambda} = 0.79$  GeV [4]. With the masses of  $\Lambda_b$  and  $\Lambda_c$  given by the Particle Data Group (PDG) [32] the upper limit of  $\omega$  is  $\omega_{\text{max}} = (1 + r^2)/2r = 1.433$ . The decay rate can then be calculated to be

$$\Gamma(\Lambda_b \rightarrow \Lambda_c \ell \bar{\nu}_\ell) = 2.12 \times 10^{-11} |V_{cb}|^2 \text{ GeV}. \quad (17)$$

The contribution of the  $1/m_Q$  corrections amount to about 10%.

Let us compare our theoretical prediction to experimental data. Recently, the DELPHI Collaboration measured the  $\Lambda_b \rightarrow \Lambda_c \ell \bar{\nu}_\ell$  branching ratio [2] as  $\mathcal{B}(\Lambda_b \rightarrow \Lambda_c \ell \bar{\nu}_\ell) = (5.0_{-0.8}^{+1.1}(\text{stat})_{-1.2}^{+1.6}(\text{sys}))\%$ . On the other hand, the branching ratio given by the PDG is  $\mathcal{B}(\Lambda_b \rightarrow \Lambda_c \ell \bar{\nu}_\ell + \text{anything}) = (9.2 \pm 2.1)\%$  [32]. An error weighted average value was given by Albertus *et al.* [33]:

$$\mathcal{B}(\Lambda_b \rightarrow \Lambda_c \ell \bar{\nu}_\ell) = (6.8 \pm 1.3)\%. \quad (18)$$

The total  $\Lambda_b$  decay width is determined by the inverse lifetime  $\tau_{\Lambda_b}^{-1}$  where we take  $\tau_{\Lambda_b} = 1.229 \pm 0.80$  ps [32]. The value of the CKM matrix element  $|V_{cb}|$  can then be extracted to be

$$|V_{cb}| = 0.041 \pm 0.004, \quad (19)$$

where the error is due to experimental uncertainties. The value of  $|V_{cb}|$  is in excellent agreement with the recent experimental determination by the DELPHI Collaboration  $|V_{cb}| = 0.0414 \pm 0.0012(\text{stat}) \pm 0.0021(\text{syst}) \pm 0.0018(\text{theory})$ , obtained from semileptonic

$B \rightarrow D^* \ell \bar{\nu}_\ell$  decays [26]. It is also in good agreement with the value of  $|V_{cb}|$  obtained from a nonrelativistic quark model calculation of the  $\Lambda_b \rightarrow \Lambda_c$  transition [33].

In conclusion, we have presented a HQET sum rule analysis for the slope parameter  $\rho^2$  of the baryonic IW function. We obtained  $\rho^2 = 1.35 \pm 0.12$  which is in good agreement with the recent DELPHI measurement and the results of a lattice calculation. When combined with the error weighted average value for the branching ratio of  $\Lambda_b \rightarrow \Lambda_c \ell \bar{\nu}_\ell$  quoted in Eq.(18), our integrated decay width including  $1/m_Q$  corrections leads to a value for  $|V_{cb}|$  in excellent agreement with a recent determination of  $|V_{cb}|$  from  $B \rightarrow D^*$  decays. One should bear in mind that  $1/m_Q^2$  corrections can decrease the decay rate by a few percent as estimated in [22]. They should be studied within HQET sum rules in future works.

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## References

- [1] N. Isgur and M. B. Wise, Phys. Lett. **B 232**, 113 (1989); E. Eichten and B. Hill, Phys. Lett. **B 234**, 511 (1990); H. Georgi, *ibid.* **B 240**, 447 (1990); A. F. Falk, H. Georgi, B. Grinstein and M. B. Wise, Nucl. Phys. **B343**, 1 (1990).
- [2] DELPHI Collaboration, J. Abdallah *et al.*, Phys. Lett. **B 585**, 63 (2004).
- [3] A. G. Grozin and O.I. Yakovlev, Phys. Lett. B **285** (1992) 254;  
E. V. Shuryak, Nucl. Phys. B **198** (1982) 83.
- [4] Y.-B. Dai, C.-S. Huang, C. Liu and C.-D. Lü, Phys. Lett. B **371** (1996) 99.
- [5] N. Isgur and M.B. Wise, Nucl. Phys. **B348**, 276 (1991); H. Georgi, *ibid.* B **348**, 293 (1991); T. Mannel, W. Roberts, and Z. Ryzak, *ibid.* B **355**, 38 (1991).
- [6] F. Hussain, J.G. Körner, M. Krämer, and G. Thompson, Z. Phys. C **51**, 321 (1991).
- [7] J.G. Körner, M. Krämer, and D. Pirjol, Prog. in Part. Nucl. Phys. **33**, 787 (1994).
- [8] For a review, see M. Neubert, Phys. Rep. **245**, 259 (1994).



- [9] A. Czarnecki and K. Melnikov, Nucl. Phys. **B505**, 65 (1997)
- [10] J.G. Körner and D. Pirjol, Phys. Lett. **B 232** (1994) 399.
- [11] M. A. Shifman, A. I. Vainshtein and V. I. Zakharov, Nucl. Phys. **B147**, 385 (1979); **B147**, 448 (1979).
- [12] For a recent review, see P. Colangelo and A. Khodjamirian, in the Boris Ioffe Festschrift *At the Frontier of Particle Physics/Handbook of QCD*, 1495-1576, edited by M. Shifman (World Scientific, Singapore, 2001).
- [13] S. Groote, J. G. Körner and O. I. Yakovlev, Phys. Rev. D **56** (1997) 3943.
- [14] B. Bagan, M. Chabab, H.G. Dosch and S. Narison, Phys. Lett. B **301** (1993) 243.
- [15] A. G. Grozin and O.I. Yakovlev, Phys. Lett. B **291** (1992) 441, hep-ph/9908364.
- [16] Y.-B. Dai, C.-S. Huang, M.-Q. Huang and C. Liu, Phys. Lett. B **387** (1996) 379.
- [17] D.-W. Wang and M.-Q. Huang, Phys. Rev. D **67** (2003) 074025.
- [18] B. König, J. G. Körner, M. Krämer and P. Kroll, Phys. Rev. D **56** (1997) 4282.
- [19] M. A. Ivanov, J. G. Körner, P. Kroll and V. E. Lyubovitskij, Phys. Rev. D **56** (1997) 348.
- [20] R.S. Marques de Carvalho, F.S. Navarra, M. Nielsen, E. Ferreira and H.G. Dosch, Phys. Rev. D **60** (1999) 034009.
- [21] H.-H. Shih, S.-C. Lee and H.-n. Li, Phys. Rev. D **61** (2000) 114002; P. Guo, H.-W. Ke, X.-Q. Li, and C.-D. Lü, hep-ph/0501058.
- [22] J. G. Körner and B. Melic, Phys. Rev. D **62** (2000) 074008.
- [23] J.-P. Lee, C. Liu and H.S. Song, Phys. Rev. D **58** (1998) 014013.  
C. Liu, Phys. Rev. D **57** (1998) 1991.
- [24] J. G. Körner, Nucl. Phys. B (Proc. Suppl.) **21** (1991) 366.
- [25] M. A. Ivanov, J. G. Körner, V. E. Lyubovitskij and A. G. Rusetsky, Phys. Rev. D **59** (1999) 074016.
- [26] DELPHI Collaboration, J. Abdallah *et al.*, Eur. Phys. J. C **33**, 213 (2004).
- [27] BELLE Coll., K. Abe *et al.*, Phys. Lett. B **526** (2004) 247.

- [28] BABAR Coll., B. Aubert *et al.*, hep-ex/0409047 (preliminary).
- [29] E. Jenkins, A. V. Manohar and M. B. Wise, Nucl. Phys. B 396 (1993) 38.
- [30] B. Blok and M. Shifman, Phys. Rev. D 47 (1993) 2949; For a detailed analysis, see: M. Neubert, Phys. Rev. D 46 (1992) 3914.
- [31] K.C. Bowler *et al.*, UKQCD Collaboration, Phys. Rev. D 57 (1998) 6948.
- [32] S. Eidelman *et al.*, Phys. Lett. B 592, 1 (2004).
- [33] C. Albertus, E. Hernández, and J. Nieves, nucl-th/0412006.